Finite Math - Fall 2016

Lecture Notes - 11/16/2016

Section 7.2 - Sets

Example 1. *Let* $A = \{3, 6, 9\}$*,* $B = \{3, 4, 5, 6, 7\}$*,* $C = \{4, 5, 7\}$*, and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$*. Find* $A \cup B$*,* $A \cap B$*,* $A \cap C$*, and* B'*.*

Example 2. *Let* $R = \{1, 2, 3, 4\}$*,* $S = \{1, 3, 5, 7\}$*,* $T = \{2, 4\}$*, and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$*. Find* $R \cup S$*,* $R \cap S$*,* $S \cap T$*, and* S'*.*

Finally, the last operation we have on sets here is to count the number of elements in a set. We will denote the number of elements in a set A by $n(A)$. In our running example of $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, we have the following facts:

$$\begin{aligned} n(A) &= 5 \\ n(B) &= 5 \\ n(A \cap B) &= 3 \\ n(A \cup B) &= 7 \\ n(A') &= 4 \\ n(A \cap B') &= 2 \\ n(\varnothing) &= 0 \end{aligned}$$

Example 3. *Let the universal set* U *be the set of positive integers less than or equal to* 100*. Let* A *be the set of multiples of* 3 *in* U*, and let* B *be the set of multiples of* 5 *in* U*.*

(a) *Find* $n(A \cap B)$*,* $n(A \cap B')$*,* $n(B \cap A')$*, and* $n(A' \cap B')$*.*

(b) *Draw a Venn diagram with circles labeled* A *and* B*, indicating the numbers of elements in the subsets of part (a).*

Solution.

(a) *The sets are*

$$A = \{3, 6, 9, 12, ..., 99\}$$

and

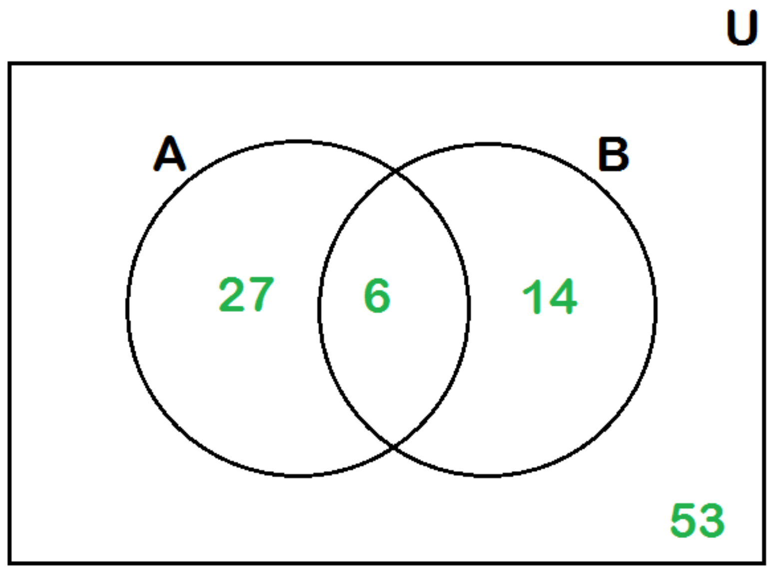
$$B = \{5, 10, 15, ..., 100\}$$

so $n(A) = 33$*,* $n(B) = 20$*. The elements in* $A \cap B$ *are multiples of both* 3 *and* 5*, so they are multiples of* 15*, i.e.,*

$$A \cap B = \{15, 30, 45, 60, 75, 90\}$$

which gives $n(A \cap B) = 6$*. Then we get* $n(A \cap B') = 33 - 6 = 27$*,* $n(B \cap A') = 20 - 6 = 14$*, and* $n(A' \cap B') = 100 - 27 - 14 - 6 = 53$*.*

(b) *The Venn diagram is*



Example 4. *Let the universal set U be the set of positive integers less than or equal to* 100*. Let A be the set of multiples of* 4 *in U, and let B be the set of multiples of* 7 *in U.*

(a) *Find $n(A \cap B)$, $n(A \cap B')$, $n(B \cap A')$, and $n(A' \cap B')$.*

(b) *Draw a Venn diagram with circles labeled A and B, indicating the numbers of elements in the subsets of part (a).*

Section 7.3 - Basic Counting Principles

Addition Principle. Suppose that there are 15 male and 20 female Physics majors at a university. How many total Physics majors are there?

Our first instinct would be to just add the total numbers and get 35 Physics majors. This, in fact, is true.

Now, suppose that every freshmen who is majoring in Chemistry is enrolled in Calculus or in History. If there are 20 freshmen Chemistry majors enrolled in Calculus and 15 freshmen Chemistry majors enrolled in History. How many total freshmen Chemistry majors are there?

We might be tempted again to just add these numbers, but what if a student is enrolled in both Calculus and History? We would then be counting that student twice! Suppose we additionally know that there are exactly 8 freshmen Chemistry majors enrolled in both Calculus and History, then we could conclude that there are $20 + 15 - 8 = 27$ total freshmen Chemistry majors. We subtract the 8 from the total because they are included in both enrollment numbers, so they would be counted twice otherwise!

We can reconcile the two situations as follows. In the first situation, let M be the set of male Physics majors and F the set of female Physics majors. Since $M \cap F = \varnothing$, we have

$$n(M \cup F) = n(M) + n(F).$$

On the other hand, in the second situation, if C is the set of these students enrolled in Calculus and H is the set of students enrolled in History, then $C \cap H \neq \varnothing$, so we must account for them. This gives

$$n(C \cup H) = n(C) + n(H) - n(C \cap H).$$

Theorem 1 (Addition Principle for Counting)**.** *For any two sets A and B,*

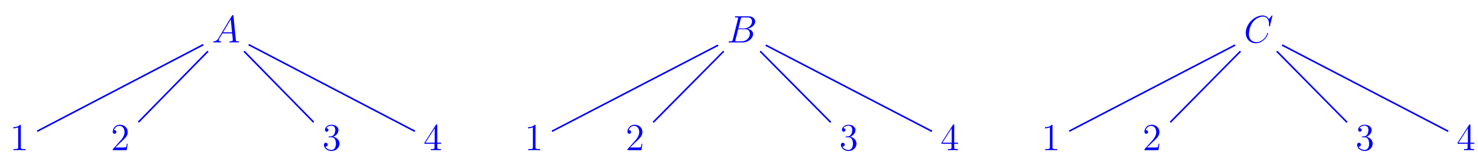
$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Example 5. *According to a survey of business firms in a certain city, 345 firms offer their employees group life insurance, 285 offer long-term disability insurance, and 115 offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?*

Multiplication Principle.

Example 6. *Suppose a store has 3 types of shirts, and in each type of shirt, they have 4 colors available. How many options are available?*

Solution. *We can write out a tree of possibilities for this. Suppose the three types of shirts are called A, B, and C, and the 4 colors are 1, 2, 3, and 4. Then the tree is*



So, there are a total of 12 possibilities in the bottom row. We can also see this by noticing that we make two choices, the first choice has 3 possibilities and the second choice has 4 possibilities. Thus there are a total of $3 \cdot 4$ overall options.

Theorem 2 (Multiplication Principle for Counting)**.**

(1) *If two operations O_1 and O_2 are performed in order, with N_1 possible outcomes for the first operation and N_2 possible outcomes for the second operation, then there are*

$$N_1 \cdot N_2$$

possible combined outcomes of the first operation followed by the second operation.

(2) *In general, if n operations $O_1, O_2, ..., O_n$ are performed in order, with possible number of number of outcomes $N_1, N_2, ..., N_n$, respectively, then there are*

$$N_1 \cdot N_2 \cdots N_n$$

possible combined outcomes of the operations performed in the given order.

Example 7. *Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?*

Solution. *36*

Example 8. *Suppose we have a list of* 8 *letters that we wish to make code words from. How many possible* 4*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Solution.

(a) *There are* 4 *options, each with* 8 *choices, so there are a total of:*

$$8 \cdot 8 \cdot 8 \cdot 8 = 8^4 = 4096$$

code words.

(b) *There are again* 4 *options. In the first option, we have all* 8 *choices; in the second options, one letter is no longer available, so there are only* 7 *choices; in the third option, two letters are no longer available, hence* 6 *choices; and the fourth option only has* 5 *choices. Thus, there are a total of:*

$$8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

code words.

(c) *There are* 4 *options. In the first option, there are all* 8 *choices; in the second option, there are now* 7 *available choices; in the third option, whatever letter was in the first slot becomes available again, so there are again* 7 *available choices; for similar reasons, the fourth option has* 7 *choices. Thus the total number of code words is*

$$8 \cdot 7 \cdot 7 \cdot 7 = 2744.$$

Example 9. *Repeat the above example, but with a list of* 10 *letters to choose from and with code words that are* 5 *letters long.*

Solution. *(a)* $100,000$*, (b)* $30,240$*, (c)* $65,610$

Example 10. *There are* 30 *teams in the MLB. Suppose a store sells both fitted and snapback baseball caps. Suppose the store carries standard and alternate versions of the fitted cap for each team, but only the standard version of the cap for the snapback cap. How many total different baseball caps do they sell?*

Solution. *This is an example where we must use a combination of adding and multiplying, since there are different number of options in the fitted and snapback cases. The total number of fitted caps are*

$$30 \cdot 2 = 60$$

and the total number of snapback caps are

$$30 \cdot 1 = 30$$

so there are a total of

$$60 + 30 = 90$$

caps at the store.